TD2

EXERCICE 1

$\pi = pq - c$

a) $\pi = qP - wl \quad q = \sqrt{l} \quad w = 1$

$P \begin{cases} \max E(\pi) \\ x \quad w = 1 \quad q = \sqrt{l} \end{cases}$

$\pi = p\sqrt{l} - l$

$E(P) = 1/2 \times 10 + 1/2 \times 30 = 5 + 15 = 20 \rightarrow$ moy

$E(\pi) = E(p)\sqrt{l} - l = 20\sqrt{l} - l \rightarrow$ univers incertain $\quad \sqrt{l} = l^{\frac{1}{2}}$

$E'(\pi)' = \frac{20}{2\sqrt{l}} - 1 = 0 \quad \frac{20}{2\sqrt{l}} = 1 \quad 20 = 2\sqrt{l} \quad 10 = \sqrt{l} \quad 100 = l \quad l > 0$

$P(p = 10) = P(p = 30) = \frac{1}{2}$

VARIANCE $= \frac{1}{m} \sum^{m} (x - \bar{x})^2$

b) Markowitz $\rightarrow U(\dot{W}_\pi) = E(\dot{W}_\pi) - KV(\dot{W}_\pi) \quad K = 0.01$

$V(\pi) = \frac{1}{2}(10\sqrt{l} - l - (20\sqrt{l} - l))^2 + 1/2(30\sqrt{l} - l - (20\sqrt{l} - l))^2$
$= 1/2(-10\sqrt{l})^2 + 1/2(10\sqrt{l})^2 =$
$= 1/2\, 100l + \frac{1}{2} 100l = 100l \longrightarrow U(l) = E(l) - K\, W(U(l))$

$U(\pi) = 20\sqrt{l} - l - 0.01(100l) \quad P\begin{cases} \max_l U(\pi) \end{cases}$

$\frac{dUl}{dl} = 0 \quad 20\frac{1}{2\sqrt{l}} - 2 = 0 \quad \frac{10}{\sqrt{l}} = 2 \quad \sqrt{l} = 5 \quad (\sqrt{l})^2 = 5^2 \quad l = 25$

c) $U(x) = \sqrt{x}$

$E(U) = P(p=10)\, U(\pi(p=10)) + P(p=30)\, U(\pi(p=30))$
$= \frac{1}{2}[\sqrt{10\cancel{l}-l}] + \frac{1}{2}[\sqrt{30\sqrt{l}-l}]$

EXERCICE 2

1) 1^{er} CAS : il vit les 2 periodes $p \rightarrow (w, \frac{w}{2})$
2^e CAS : il vit 1 seule periode $1-p \rightarrow (w, 0)$

2) $S(w_1, w_2) = \ln w_1 + \frac{\ln w_2}{(1+r)}$

$E(S(w_1\ w_2)) = p(\text{vie}) \times U(\text{vie}) + p(\text{mort}) \times U(\text{mort})$
$= p[\ln w + \frac{\ln(w/2)}{(1+r)}] + (1-p)[\ln w]$
$= \ln w + p\frac{\ln w - \ln 2}{(1+r)} = \ln(w) + p\frac{\ln(w/2)}{1+r}$

3) 1er CAS : il vit les 2 periodes $p \rightarrow (w - Bi\ ;\ \frac{w}{2} + i)$
2^e CAS : il vit ul seule periode $p \rightarrow (w - Bi\ ,\ 0)$

$E(S(w_1, w_2)) = \ln(w - Bi) + p\frac{(\ln\frac{w}{2} + i)}{1+r}$

4) $\frac{\partial E(S(w_1, w_2))}{\partial i} = 0 \quad \frac{-B}{w - Bi} + \frac{p}{1+r} \times \frac{1}{\frac{w}{2} + i} = 0 \iff \frac{-B}{w - Bi} + \frac{p}{1+r}\frac{2}{w + 2i} = 0$

$\frac{1}{\frac{w+2i}{2}}$

$\iff \frac{p}{(1+r)(\frac{w}{2} + i)} = \frac{B}{w - Bi} \iff \frac{p}{(1+r)(\frac{w}{2}+i)} w - Bi = B$

$\iff \cancel{\frac{p}{(1+r)(\frac{w}{2}+i)} \frac{w - Bi((1+r)(\frac{w}{2}+i))}{(1+r)(\frac{w}{2}+i)} = B}$

$\Leftrightarrow \cancel{\frac{p}{(1+r)(\frac{w}{2}+i)} w - Bi = B \Leftrightarrow p(w-Bi) = B(1+r)(\frac{w}{2}+i)}$

$2pw - 2pBi = B(1+r)(2i+w) \Leftrightarrow 2pw - Bw(1+r) = i[2B(1+r) + 2pB]$

$i^* = \frac{2p - B(1+r)w}{2B[1+r+p]}$

5) $i^* \geq 0 \quad 2p - B(1+r) \geq 0 \quad 2p \geq B(1+r) \quad B \leq \frac{2p}{1+r}$

6) demande de pension ~~recette~~ retraite est croissante en w
7) decroissante en B 8) croissante en p
9) decroissant en r

EXERCICE 3

$U(X) = \left(\frac{X}{10}\right)^2 \quad E(U(A)) = E(U(B))$

$0.5 \frac{110^2}{100} + \frac{130^2}{100} 0.5 = p\left(\frac{90}{10}\right)^2 + (1-p)\left(\frac{150}{10}\right)^2$

$0.5\, U(110) + 0.5\, U(130) = pU(90) + (1-p)U(150) \quad p = \frac{80}{144} = \frac{5}{9}$

EXERCICE 4 :

K cases $A_1, \dots, A_K$

$P(A_1) = \dots = P(A_K) = \frac{1}{K} \quad m = x_1 + \dots x_K = \sum_1^K x_i$

a) $E(x) = \sum_{i=1}^K P(A_i) dx_i = P(A_1) dx_1 + \dots + P(A_K) dx_K$

$= \frac{1}{K} dx_1 + \dots + \frac{1}{K} dx_K = \frac{1}{K} d(\underbrace{x_1 + \dots + x_K}_{=m}) \quad E(x) = \frac{dm}{K} \quad d = \lambda$

$V(x) = \frac{1}{K} \sum_{i=1}^K [dx_i - E(x)]^2 = \frac{1}{K} \sum_{i=1}^K [dx_i - \frac{dm}{K}]^2 = \frac{\lambda^2}{K} \sum_{i=1}^K [x_i - \frac{m}{K}]^2$

b) $\begin{cases} \min V(x) \\ x_1 \dots x_K \end{cases} \quad \begin{cases} \min \frac{\lambda^2}{K} \sum_i^K (x_i - \frac{m}{K})^2 \\ x_1 \dots x_K \end{cases} \quad \begin{cases} \min \sum_i^K (x_i - \frac{m}{K})^2 \\ x_1 \dots x_K \end{cases}$

$\forall x_i, i = 1, \dots, K$

$\min_{x_i} (x_i - \frac{m}{K})^2$ sachant $x^2 \geq 0 \ \forall X \in \mathbb{R}$

$\Leftrightarrow x_i - \frac{m}{K} = 0 \quad \forall x_i \ i = 1, \dots, K \quad x_i = \frac{m}{K}$

c) $U(w) = \ln w \quad \begin{cases} \max E(U(x_1 \dots x_K)) \\ x_1 \dots x_K \ \text{sc} \ x_1 + x_2 + \dots x_K = m \end{cases}$

$\begin{cases} \max \frac{1}{K} \sum_i^K \ln(\lambda x_i) \\ \text{sc} \ m = x_1 + x_2 \dots x_K \end{cases}$

$\mathcal{L} = \frac{1}{K} \sum_{i=1}^K \ln(\lambda x_i) + \mu[\sum_{i=1}^K x_i - m] = \frac{1}{K} \ln(\lambda x_1) + \dots \frac{1}{K} \ln(\lambda x_K) + \mu[x_1 + \dots x_K - m]$?

CPO : (1) $\frac{d\mathcal{L}}{dx_1} = \frac{1}{K} \frac{\lambda}{\lambda x_1} + \mu = 0$ (K) $\frac{d\mathcal{L}}{dx_K} = 0 \quad \frac{1}{K} \frac{\lambda}{\lambda x_K} + \mu = 0$

(K+1) $\frac{d\mathcal{L}}{d\mu} = 0 \quad x_1 + x_2 + \dots x_K = m$

$\left.\begin{array}{l} (1) \ \frac{\lambda 1}{\lambda K x_1} = -\mu \\ (K) \ \frac{1}{K x_K} = -\mu \end{array}\right\} x_1 = x_K$ donc $x_i^* = x_2^* = \dots = x_K^*$ donc $= \frac{m}{K}$

Autre Méthode

$$x_K = m - (x_1 + x_2 + \dots + x_K)$$

$$U = \frac{\ln(\lambda x_1)}{x} + \dots \frac{\ln(\lambda x_K)}{K} + \frac{1}{K} \ln(\lambda [m - (x_1 + x_2 + \dots + x_K]))$$

$$\max_{(x_1 \dots x_K)} \frac{1}{K} \sum_{i=1}^{K} \ln(\lambda x_i)$$

$$\text{sc} \quad m = x_1 + x_2 + \dots x_K$$

TD 3

EXERCICE 3

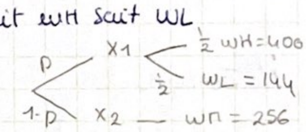
a) $L_{x1} (w_H, w_L \; ; \; \frac{1}{2}, \frac{1}{2})$ $P(w=400)$ $P(w=200)$ $L_{x2} = (w_M \; ; \; 1)$ → eq certain

b) $E(L) = \frac{1}{2} \times 400 + \frac{1}{2} \times 200 = 300 > 280$ → averse au risque

c) $L_C = (\frac{p}{2}, \frac{p}{2}, 1-p \; ; \; 400, 144, 256)$ $P = X_1$ = soit w_H soit w_L

$P(w=400) \to U(400) \dots$ $1-p = x_2$

d) $U(x) = \sqrt{x}$ $E(U(x)) = \frac{p}{2} \times \sqrt{400} + \frac{p}{2} \sqrt{144} + (1-p)\sqrt{256}$



$E(U(x)) = \frac{20p}{2} + \frac{12p}{2} + (1-p)16 = 10p + 6p + 16 - 16p = 16$

$\sqrt{x} = 16$ $x = 16^2$ $x = 256$ OR $256 = w_M$ ⇒ salaire certain

on choisit X_2 car choisit pas X_1 → Averse au risque

où ya de l'incertitude, X_2 est certain, n'aime pas le risque

e) $E[U(x)] = \frac{p}{2} \times 400^2 + \frac{p}{2} \times 144^2 + (1-p) 256^2$

$E[U(x)] = 24\,832p + 65\,536$ → croissant en p donc goût pour risque

→ on se dirigera vers X_1

EXERCICE 4 :

a) Si X neutre au risque alors il achète la voiture si :

$E[w + 2000q - p] \geqslant E[w]$ $p \leqslant 2000 E[q]$

OR $E(q) = \frac{1}{2} \times 2 + \frac{1}{2} \times 8 = 5$ $p \leqslant 2000 \times 5 \leqslant 10\,000$ → prix achat si neutre

b) $U(x) = \ln(x) \Leftrightarrow \frac{1}{2} \ln(w - p + 2000 \times 2) + \frac{1}{2} \ln(w - p + 2000 \times 8) = \ln(w)$

$\Leftrightarrow \frac{1}{2} \ln(100\,000 - p + 2000 \times 2) + \frac{1}{2} \ln(100000 - p + 2000 \times 8) = \ln(100\,000)$

$\Leftrightarrow \ln(104\,000 - p) + \ln(116\,000 - p) = 2\ln(100\,000)$

$\ln[(104\,000 - p)(116\,000 - p)] = \ln(100\,000)^2$ $p^* \simeq 9820$

↳ $2064 - \cancel{210}p + p^2 = 0$

↳ $\Delta = -220^2 - 4(1)(2064$

c) prime $= 10\,000 - 9820 = 180$