c) $\frac{da^*}{dw} = \ominus \frac{\lambda > 0}{1-\lambda^2 > 0} \Big\} < 0 \quad \forall \lambda \in [0;1[$

EXERCICE 4

alea moral = conseq d'une asymetrie d'info

TD 7

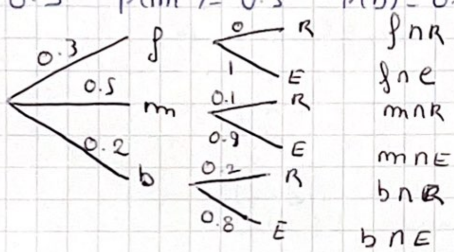
EXERCICE 1 : on cherche $P(f|R)$, $f(m|R)$, $P(b|R)$

$P(R|f) = 0.8$ $\quad P(R|m) = 0.9$ $\quad P(R|b) = 1$

FORMULE DE BAYES : $P(f|R) = \dfrac{P(R|f)\,P(f)}{P(R|f)P(f) + P(R|m)P(m) + P(R|b)P(b)}$

$= 0.2696$

$P(f) = 0.3$ $\quad P(m) = 0.5$ $\quad P(b) = 0.2$



$P(f|R) = \dfrac{P(f\cap R)}{P(f\cap R) + P(m\cap R) + P(b\cap R)}$

EXERCICE 2 :

a) aucune informat° $\quad$ R : reforme $\quad \bar{R}$: pas de reforme

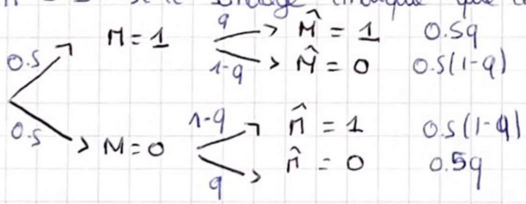
R = 1 si reelection et 0 sinon

$E(R|R) = 0.5\times 1 + 0.5\times 0 = 0.5$ $\quad E(R|\bar{R}) = 0.6\times 1 + 0.4\times 0 = 0.6$

b) sondage : $\frac{1}{2} < q < 1$

$M = 1$ si la majorité en faveur (0 sinon)

$\hat{M} = 1$ si le sondage indique que les electeurs sont en faveur (0 sinon)



$P(M=1|\hat{M}=0) = \dfrac{0.5(1-q)}{0.5(1-q)+0.5q} = 1-q$

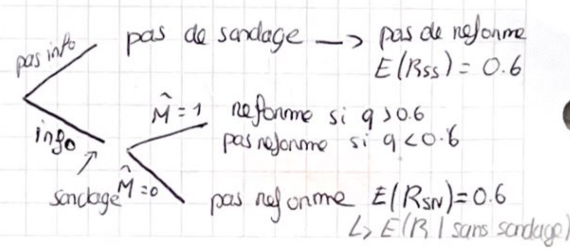
$\hookrightarrow \dfrac{P(M=1\cap \hat{M}=0)}{P(\hat{M}=0)}$

$P(M=0|\hat{M}=0) = \dfrac{0.5q}{0.5(1-q)+0.5q} = q$

Si le sondage revient positif :

$P(M=1|\hat{M}=1) = \dfrac{0.5q}{0.5q+0.5(1-q)} = q$

$P(M=0|\hat{M}=1) = \dfrac{0.5q(1-q)}{0.5q+0.5(1-q)} = 1-q$



$\rightarrow R=1$

negatif : $E(R \mid \hat{M}=0, R) = 1 \times P(M=1 \mid \hat{M}=0) + 0 \times P(M=0 \mid \hat{M}=0) = 1-q$

positif : $E(R \mid \hat{M}=1, R) = 1 \times P(M=1 \mid \hat{M}=1) + 0 \times P(M=0 \mid \hat{M}=1) = q$

$1-q < 0.6$ car $q > \frac{1}{2}$ donc pas de reforme

Valeur du sondage :

$$V = E(R \mid \text{sondage}) - E(R \mid \text{pas de sondage}) = P(\overset{\oplus}{\hat{M}=1})E(R_{SP}) + P(\overset{\ominus}{\hat{M}=0})E(R_{SN}) - 0.6$$

$$V = 0.5q + 0.5 \times 0.6 - 0.6 = 0.5(q - 0.6) = 0.5q - 0.3$$

TD 9

EXERCICE 1 $\quad i \in [0,1] \quad$ frais medicaux : $100i$

a) $i = 0.6$: $\mathcal{L} = ((150 - 100 \times 0.6, 150) ; (\frac{1}{2}, \frac{1}{2}))$

$= (90, 150) ; (\frac{1}{2}, \frac{1}{2})$

$E(W_{\mathcal{L}}) = \frac{1}{2} 90 + \frac{1}{2} 150 = 120$

$E(U) = \frac{1}{2} \ln(90) + \frac{1}{2} \ln(150) = 4.76$

Equivalent certain : EC tel que : $E(U) = U(EC) \Leftrightarrow 4.76 = \ln(EC)$

$\Leftrightarrow EC = \exp(4.76) \simeq 116$

Prime de risque : $120 - 116 = 4$

b) $\forall i \in [0,1]$ $\quad \mathcal{L} = ((150 - 100i, 150) ; (\frac{1}{2}, \frac{1}{2}))$

$E(W_{\mathcal{L}}) = \frac{1}{2}(150 - 100i) + \frac{1}{2}(150)$

$E(U) = \frac{1}{2} \ln(150 - 100i) + \frac{1}{2} \ln(150)$

Equivalent certain EC tel que : $E(U) = U(EC) \Leftrightarrow \frac{1}{2} \ln(150 - 100i) + \frac{1}{2} \ln(150) = \ln(EC)$

$EC = \exp\left(\frac{1}{2} \ln(150 - 100i) + \frac{1}{2} \ln 150\right)$

prime de risque : $E(W) - EC$. $\frac{1}{2}(150 - 100i) + \frac{1}{2}(150) = EC$

c) Prime assurance $\Leftrightarrow P(\text{malade}) \times E(\text{indemnité})$

$\Leftrightarrow P(\text{malade}) \times E(\text{dommage}) = \frac{1}{2} E(100i) = 50E(i) = 25$

$E(i) = \int_0^1 x\,dx = \frac{1}{2} \rightarrow \frac{b-a}{2} \quad \frac{1-0}{2} = \frac{1}{2}$

$E(X) = \int x f(x)\,dx$

$E(X \mid B) = \frac{1}{P(B)} \int x f(x)\,dx$

d) choix de l'individu

indifférent si : $\cancel{\frac{1}{2} \ln(125) + \frac{1}{2} \ln(150) + \frac{1}{2} \ln(150 - 100i)}$

$150 - 25$ (ok)

$$\underbrace{\frac{1}{2} \ln(125) + \frac{1}{2} \ln(125)}_{\text{s'assurer}} = \underbrace{\frac{1}{2} \ln(150) + \frac{1}{2} \ln(150 - 100i)}_{\text{pas s'assurer}}$$

$(150 - 25 - 100i + 100i)$

$i = 0.46$

non $\quad$ assurance

0 $\quad$ 0.46 $\quad$ 1 $\quad i$