TD 1

Exercice 1

TMS a) $U(x_1, x_2) = x_1 x_2$

① Max $U(x_1, x_2) = x_1 x_2$
sc $R = p_1 x_1 + p_2 x_2$

② $\mathcal{L}(q) = u(q) + \lambda(x - pq)$
$= x_1 x_2 + \lambda(R - p_1 x_1 - p_2 x_2)$

③ CPO : $\frac{d\mathcal{L}(x_1, x_2, \lambda)}{dx_1} = 0$; $\frac{d\mathcal{L}(x_1, x_2, \lambda)}{dx_2} = 0$; $\frac{d\mathcal{L}}{d\lambda}$

$\begin{cases} x_2 - \lambda p_1 = 0 \\ x_1 - \lambda p_2 = 0 \\ R - p_1 x_1 - p_2 x_2 = 0 \end{cases}$ $\begin{cases} x_2 = \lambda p_1 & ① \\ x_1 = \lambda p_2 & ② \\ R = p_1 x_1 + p_2 x_2 & ③ \end{cases}$

④ $\frac{1}{2}$ $\frac{x_2}{x_1} = \frac{p_1}{p_2} \Rightarrow x_2 = x_1 \frac{p_1}{p_2}$

$R = p_1 x_1 + p_2 \left(\frac{p_1}{p_2} x_1\right)$
$\Leftrightarrow R = 2 p_1 x_1$

$x_1^* = \frac{R}{2p_1}$ $x_2^* = \frac{R}{2p_2}$

TMS b) $U(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$

① Max $U(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$
sc $R = p_1 x_1 + p_2 x_2$

② $\mathcal{L}(x) = U(x) + \lambda(R - px)$
$= x_1^{\alpha} x_2^{1-\alpha} + \lambda(R - (p_1 x_1 + p_2 x_2))$

③ CPO : $\frac{d\mathcal{L}(x_1, x_2, \lambda)}{dx_1} = 0$; $\frac{d\mathcal{L}(x_1, x_2, \lambda)}{dx_2} = 0$; $\frac{d\mathcal{L}}{d\lambda}$

$\begin{cases} \alpha x_1^{\alpha-1} x_2^{1-\alpha} = \lambda p_1 & ① \\ (1-\alpha) x_2^{-\alpha} x_1^{\alpha} = \lambda p_2 & ② \\ R = p_1 x_1 + p_2 x_2 & ③ \end{cases}$

$\frac{1}{2}$ $\frac{\alpha x_1^{\alpha-1} x_2^{1-\alpha}}{(1-\alpha) x_2^{-\alpha} x_1^{\alpha}} = \frac{p_1}{p_2} \Rightarrow \frac{\alpha}{1-\alpha} x_1^{-1} x_2 = \frac{p_1}{p_2}$ ($\to \frac{1}{x_1}$)

$\Rightarrow \frac{\alpha}{1-\alpha} \frac{x_2}{x_1} = \frac{p_1}{p_2} \Rightarrow$ isole x_2 $x_2 = \frac{p_1(1-\alpha)}{p_2 \alpha} x_1$

remplace ds R $R = p_1 x_1 + p_2 \left(\frac{p_1(1-\alpha)}{p_2 \alpha} x_2\right)$
$= p_1 x_1 \left(1 + \frac{1-\alpha}{\alpha}\right) = p_1 x_1 \frac{1}{\alpha}$ $R = \frac{p_1 x_1}{\alpha}$

$x_1^* = \frac{R\alpha}{p_1}$ $x_2^* = \frac{R(1-\alpha)}{p_2}$

(x_1 apporte m̂ utilité que x_2 on va choisir le - cher)

substituts parfait c) $U(x_1, x_2) = x_1 + x_2$

① Max $U(x_1, x_2) = x_1 + x_2$
sc $R = p_1 x_1 + p_2 x_2$

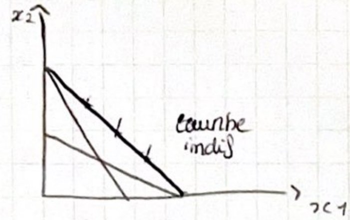
$\mathcal{L}(x) = U(x) + \lambda(R - px)$

CPO $\frac{d\mathcal{L}}{dx_1} = 1 - \lambda p_1 = 0$ $1 = \lambda p_1$ ①

$\frac{d\mathcal{L}}{dx_2} = 1 - \lambda p_2 = 0$ $1 = \lambda p_2$ ②

$\frac{d\mathcal{L}}{d\lambda} = R - p_1 x_1 - p_2 x_2$ ③

$\frac{1}{2} = \frac{1 p_1}{1 p_2} = \frac{p_1}{p_2} \rightarrow 1 = \frac{p_1}{p_2}$



$U = x_1 + x_2 = \bar{u}$

$x_2 = \bar{u} - x_1$

$p_1 > p_2 \quad x_1^* = 0 \quad x_2^* = \frac{R}{p_2}$

$p_1 < p_2 \quad R = p_1 x_1$ car $x_2 = 0 \quad x_1^* = \frac{R}{p_1} \quad x_2^* = 0$

$p_1 = p_2 \quad R = p_1 x_1 + p_2 x_2 \quad R = p_1(x_1 + x_2)$ ou $p_2(x_1+x_2)$

$x_1 + x_2 = \frac{R}{p_1}$ ou $x_1 + x_2 = \frac{R}{p_2}$

comp parfait

$p_1 > p_2 \quad R = p_2 x_2$ car $x_1 = 0 \quad x_1^* = 0 \quad x_2^* = \frac{R}{p_2}$

d) $U(x_1, x_2) - \min\{x_1, x_2\}$

Max $U(x_1, x_2)$

$R = p_1 x_1 + p_2 x_2$

$x_1 = x_2$

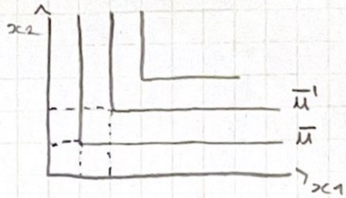
$R = p_1 x_1 + p_2 x_1$

$R = x_1 (p_1 + p_2)$

$x_1 = \frac{R}{p_1 + p_2}$

$R = x_2 (p_1 + p_2)$

$x_2 = \frac{R}{p_1 + p_2}$



e) $U(x_1, x_2) - (x_1^p + x_2^p)^{1/p}$

$\mathcal{L} = (x_1^p + x_2^p)^{\frac{1}{p}} + \lambda (R - p_1 x_1 + p_2 x_2)$

$U(x_1, x_2) = (x_1^p + x_2^p)^{1/p}$

sc $R = p_1 x_1 + p_2 x_2$

$\frac{d\mathcal{L}}{dx_1} = 0 \quad \underbrace{\frac{1}{p}}_{\alpha} \underbrace{(p x_1^{p-1})}_{u'} \underbrace{(x_1^p + x_2^p)^{\frac{1}{p}-1}}_{u^{\alpha-1}} = \lambda p_1$ ①

$\frac{d\mathcal{L}}{dx_2} = 0 \quad \frac{1}{p} (p x_2^{p-1}) (x_1^p + x_2^p)^{\frac{1}{p}-1} = \lambda p_2$ ②

$(U(x)^\alpha)' = \alpha u' u^{\alpha - 1}$

$\frac{\cancel{\frac{1}{p}} (\cancel{p} x_1^{p-1}) \cancel{(x_1^p + x_2^p)^{\frac{1}{p}-1}}}{\cancel{\frac{1}{p}} (\cancel{p} x_2^{p-1}) \cancel{(x_1^p + x_2^p)^{\frac{1}{p}-1}}} = \frac{\cancel{\lambda} p_1}{\cancel{\lambda} p_2}$

$\frac{x_1^{p-1}}{x_2^{p-1}} = \frac{p_1}{p_2} \Leftrightarrow \frac{x_1}{x_2} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{p-1}}$

$x_1 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{1-p}} x_2 \qquad x_1 = p_2^{\frac{1}{1-p}} \, p_1^{\frac{1}{p-1}} x_2$

$\hookrightarrow$ pcq $-\frac{1}{p-1}$

$\hookrightarrow p_2 \times p_2^{\frac{1}{1-p}}$

dans la CB : $R = p_1 x_1 + p_2 x_2$

$R = \underline{p_1 p_2^{\frac{1}{1-p}} p_1^{\frac{1}{1-p}}} x_2 + p_2 x_2$

$R = p_1^{\frac{1+p-1}{p-1}} p_2^{\frac{1}{1-p}} x_2 + p_2 x_2$

$R = p_1^{\frac{p}{p-1}} p_2^{\frac{1}{1-p}} + p_2] x_2$

$R = [p_1^{\frac{p}{p-1}} + p_2^{\frac{1-p-1}{1-p}}] p_2^{\frac{1}{1-p}} x_2$

$x_2^* = \frac{p_2^{p-1} \cdot R}{p_1^{\frac{p}{p-1}} + p_2^{\frac{p}{p-1}}}$

$\frac{1}{p} (p x_2^{p-1}) (x_1^p + x_2^p)^{\frac{1}{p}-1} = \lambda p_2$ ②

$\nearrow p_A \searrow x_2$

si $p > 1$ complementaire

si $p < 1$ substituables

f) $U(x_1, x_2) = (1 + x_1)(1 + x_2)$

Max $(1 + x_1)(1 + x_2)$

$R = p_1 x_1 + p_2 x_2$

$\mathcal{L}(x_1, x_2, \lambda) = (1 + x_1)(1 + x_2) + \lambda (R - p_1 x_1 - p_2 x_2)$

CPO : $\frac{d\mathcal{L}}{dx_1} = 0 \quad , \quad \frac{d\mathcal{L}}{dx_2} = 0 \quad \frac{d\mathcal{L}}{d\lambda} = 0$

$1 + x_2 = \lambda p_1$ $\qquad \frac{1}{2} \frac{1+x_2}{1+x_1} = \frac{\lambda p_1}{\lambda p_2}$

$1 + x_1 = \lambda p_2$

$R = p_1 x_1 + p_2 x_2$ $\qquad x_2 = \frac{p_1}{p_2}(1+x_1) - 1$

Dans la CB $\quad R = p_1 x_1 + p_2 x_2 = p_1 x_1 + p_2 \left[\frac{p_1}{p_2} (1 + x_1) - 1 \right]$

$= p_1 x_1 + p_1 + p_1 x_1 - p_2$

$R + p_2 - p_1 = 2 p_1 x_1$ $\qquad x_1^* = \frac{R + p_2 - p_1}{2 p_1}$

$x_2^* = \frac{R + p_1 - p_2}{2 p_2}$

OK

3) 2 biens : x_1, x_2 $\quad R = p_1 x_1 + p_2 x_2$ $\quad \nearrow R$ telle que $R' > R$

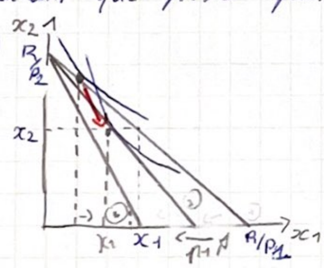
Nv panier à l'eq $R' = p_1 x_1' + p_2 x_2'$

$R' > R \quad p_1 x_1' + p_2 x_2' > p_1 x_1 + p_2 x_2$

$p_1 (x_1' - x_1) + p_2 (x_2' - x_2) > 0 \quad \rightarrow$ La diff

sachant que $p_1 > 0$ $\quad p_2 > 0$ $\quad$ donc on a $\underline{x_1' - x_1 > 0}$ $\quad \underline{x_2' - x_2 > 0}$

4)



CB : $R = p_1 x_1 + p_2 x_2$

$x_2 = \frac{R}{p_2} - \frac{p_1}{p_2} x_1 \rightarrow$ f° affine CD : -

Bien GIFFEN : $p_1 \nearrow$ $x_1 \nearrow$

5) ① independants $x_1^* = \frac{R}{2p_1}$ $\quad$ ② independants $\quad$ ③ parf substituables

④ parf complémentaires $\quad$ ⑤ comp si $p > 1$ substitu si $p < 1$ $\quad$ ⑥ substituables