TD4

EXERCICE 1 :

$\ln(a) + \ln b = \ln ab$
$\ln a - \ln b = \ln \frac{a}{b}$

a) $W : \left[\left(\frac{1}{3} ; \frac{1}{3} ; \frac{1}{3}\right) ; (101, 110, 200)\right]$

b) prime de risque : $u_1(x) = \ln(x)$ $\quad u_2(x) = \sqrt{x}$

$E(u_1) = \frac{1}{3} \ln(101) + \frac{1}{3} \ln(110) + \frac{1}{3} \ln(200) = \frac{1}{3} \ln(101 \times 110 \times 200)$

$= \ln(101 \times 110 \times 200)^{\frac{1}{3}}$

Eq Certain : $\ln(x) = \ln[(101 \times 110 \times 200)]^{1/3} = \frac{1}{3} \ln 101 + \frac{1}{3} \ln 110 + \frac{1}{3} \ln 200$

$x = (101 . 110 . 200)^{1/3} = 130.49$

$E(w) = \frac{101 + 110 + 200}{3} = \frac{411}{3} = 137$ $\quad$ Prime de risque 1 : $E(W) - EC_1 = 137 - 130.49 = 6.51$

pour u_2 : $x = 133.63$ $\quad$ Prime de risque 2 : $E(W) - EC = 137 - 133.63 = 3.37$

1 a ⊕ d'aversion au risque.

c) Coeff d'aversion au risque : $e = -\frac{u''(x)}{u'(x)}$ = CARR (absolu)

$\gamma = x \frac{u''(x)}{u'(x)}$ = CARA (relatif) $= xe$

$e_1 = -\frac{1}{x^2} / \frac{1}{x} = \frac{1}{x}$ $\quad u'_1 = \frac{1}{x}$ $\quad u''_1 = -\frac{1}{x^2}$ $\quad e_1 = \frac{1}{x}$

$u'_2(x) = \frac{1}{2\sqrt{x}} = \frac{1}{2} x^{-1/2}$ $\quad u''_2(x) = -\frac{1}{4} x^{-3/2} = \frac{1}{2}\left(-\frac{1}{2}\right) x^{-\frac{1}{2} \times 1}$

$e^2 = \frac{\frac{1}{2} \frac{1}{2} x^{-\frac{1}{2}-1}}{\frac{1}{2} \times x^{-\frac{1}{2}}} = \frac{1}{2} x^{-1} \Rightarrow e_2 = \frac{1}{2x}$

$\forall x$, $e_1 > e_2$ donc 1 est plus averse au risque.

Exercice 2 :

On sait que $u(w) = f(v(w))$ $\quad f' > 0 \quad f'' < 0$
On veut montrer que $A_u > A_v$
avec A_i la mesure d'Arrow Pratt de i

$A_u = -\frac{u''(w)}{v'(w)} \cdot u'(w)$

$u''(w) = \underbrace{f''(v(w))\, v'(w)^2}_{\hookrightarrow\, f'(v(w))' \, v'(w) \,=\, v'(w) \times f'(v(w)) \times v'(w)} + f'(v(w))\, v''(w)$

$A_u = -\frac{f''(v(w))\, v'(w)^2 + f'(v(w))\, v''(w)}{f'(v(w)) \times v'(w)} = -\left[\frac{f''(v(w))\, v'(w)}{f'(v(w))} + \frac{v''(w)}{v'(w)}\right]$?

$A_u = -\frac{f''(v(w))\, v'(w)}{f'(v(w))} \underbrace{- \frac{v''(w)}{v'(w)}}_{A_v}$ $\iff A_u > A_v \iff -\frac{f''(v)v'}{f'(v)} > 0$

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$f'' < 0$ $f' > 0$ $v' > 0$ $\}$ > 0 donc $A_u > A_v$

EXERCICE 5 :

$R_A : 0.05(-0.02) + 0.15(0.01) + 0.60(0.15) + 0.2(0.15) = 0.1175$
$R_B : 0.125$

$V(A) = 0.05(-0.02 - 0.1175)^2 + 0.15(-0.1 - 0.1175)^2 + 0.6(0.15 - 0.1175)^2$
$+ 0.2(0.15 - 0.1175)^2$

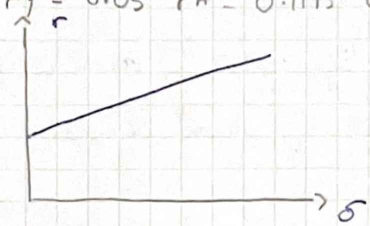
$\sqrt{V(A)} = 0.065 = \sigma_A$ $\qquad \sigma_B = 0.1392$

$E(R_p) = (1-\lambda)r + \lambda R \qquad \sigma(R_p) = \lambda\,\sigma(R)$

$E(R_p) = r + \frac{R - r}{\sigma(R)}\,\sigma(R_p)$

$r = r_f + \frac{r_A - r_f}{\sigma_A}\,\sigma \qquad r_f = 0.05 \quad r_A = 0.1175 \quad \sigma_A = 0.065$

$r = 0.05 + 1.04\,\sigma$



TD6

Exercice ~~6~~ 2

① Espérance d'utilité :

"RICHESSE"

- CAS 1 accident ($x = 2$) : $(V - V+) - p + \alpha X \Rightarrow -(1+\lambda)E(I) + \alpha X$
- CAS 2 pas accident ($x = 0$) : $v - p \Rightarrow v - (1+\lambda)E(I)$

$E(I) = E(aX) = \alpha E(X) = \alpha \qquad E(X) = \frac{1}{2} \times 2 + \frac{1}{2} \times 0 = 1$

Cas 1 : $2a - (1+\lambda)a \qquad$ Cas 2 : $2 - (1+\lambda)a$

$* \; E(U) = \frac{1}{2}\,u(2a - (1+\lambda)a) + \frac{1}{2}\,U(2 - (1+\lambda)a$ $\qquad$ avec $U(x) = \ln(x)$

$\bullet \; E(U) = \frac{1}{2}\ln(2a - (1+\lambda)a) + \frac{1}{2}\ln(2 - (1+\lambda)a)$

b) ① $\left\{ \max_{\{a\}} E(U) \right\}$ $\qquad$ ② CPO : $\frac{dE(U)}{da} = 0$ $\qquad 2$

$\Rightarrow \frac{1}{2}\,\frac{2 - (1+\lambda)}{2a - (1+\lambda)a} + \frac{1}{2}\,\frac{-(1+\lambda)}{2 - (1+\lambda)a} = 0 \quad \Rightarrow \frac{2 - (1+\lambda)}{2a - (1+\lambda)a} = \frac{1+\lambda}{2 - (1+\lambda)a}$

$\Rightarrow 2 - 2\lambda - (1+\lambda)(1-\lambda)a = 2a(1+\lambda) - a(1+\lambda)^2$

$\Rightarrow 2 - 2\lambda = a(1 - \lambda + \lambda + \lambda^2 + 2 + 2\lambda - 1 - 2\lambda - \lambda^2)$

$\Rightarrow 2(1-\lambda) = 2a(1-\lambda^2)$

$\Rightarrow (1-\lambda) = a(1-\lambda^2) \Leftrightarrow a = \frac{1-\lambda}{1-\lambda^2} \qquad 1 - \lambda^2 = (1-\lambda)(1+\lambda) \qquad a^* = \frac{1}{1+\lambda}$