$f'' < 0 \quad f' > 0 \quad v' > 0 \Big\} > 0$ donc $A_u > A_v$

EXERCICE 5 :

$R_A : 0.05(-0.02) + 0.15(0.01) + 0.60(0.15) + 0.2(0.15) = 0.1175$
$R_B : 0.125$

$V(A) = 0.05(-0.02 - 0.1175)^2 + 0.15(-0.1 - 0.1175)^2 + 0.6(0.15 - 0.1175)^2$
$+ 0.2(0.15 - 0.1175)^2$

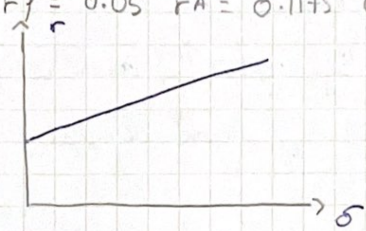
$\sqrt{V(A)} = 0.065 = \sigma_A \qquad \sigma_B = 0.1392$

$E(R_p) = (1-\lambda) r + \lambda R \qquad \sigma(R_p) = \lambda \sigma(R)$

$E(R_p) = r + \dfrac{R - r}{\sigma(R)} \sigma(R_p)$

$r = r_f + \dfrac{r_A - r_f}{\sigma_A} \sigma \qquad r_f = 0.05 \quad r_A = 0.1175 \quad \sigma_A = 0.065$

$r = 0.05 + 1.04\,\sigma$



TD6

Exercice 2

① Espérance d'utilité :

"RICHESSE"

- CAS 1 accident ($x = 2$) : $(V - V+) - p + \alpha X \Rightarrow -(1+\lambda)E(I) + \alpha X$
- CAS 2 pas accident ($x = 0$) : $V - p \Rightarrow V - (1+\lambda)E(I)$

$E(I) = E(aX) = \alpha E(X) = \alpha \qquad E(X) = \frac{1}{2} \times 2 + \frac{1}{2} \times 0 = 1$

Cas 1 : $2a - (1+\lambda)a \qquad$ Cas 2 : $2 - (1+\lambda)a$

* $E(U) = \frac{1}{2} U(2a - (1+\lambda)a) + \frac{1}{2} U(2 - (1+\lambda)a$ avec $U(x) = \ln(x)$

• $E(U) = \frac{1}{2} \ln(2a - (1+\lambda)a) + \frac{1}{2} \ln(2 - (1+\lambda)a)$

b) ① $\left\{ \underset{\{a\}}{Max}\ E(U) \right\}$ ② CPO : $\dfrac{dE(U)}{da} = 0$ 2

$\Rightarrow \frac{1}{2} \dfrac{2 - (1+\lambda)}{2a - (1+\lambda)a} + \frac{1}{2} \dfrac{-(1+\lambda)}{2 - (1+\lambda)a} = 0 \Rightarrow \dfrac{2 - (1+\lambda)}{2a - (1+\lambda)a} = \dfrac{1+\lambda}{2 - (1+\lambda)a}$

$\Rightarrow 2 - 2\lambda - (1+\lambda)(1-\lambda)a = 2a(1+\lambda) - a(1+\lambda)^2$
$\Rightarrow 2 - 2\lambda = a(1 - \lambda + \lambda + \lambda^2 + 2 + 2\lambda - 1 - 2\lambda - \lambda^2)$
$\Rightarrow 2(1-\lambda) = 2a(1 - \lambda^2)$
$\Rightarrow (1 - \lambda) = a(1 - \lambda^2) \iff a = \dfrac{1-\lambda}{1-\lambda^2} \qquad 1 - \lambda^2 = (1-\lambda)(1+\lambda) \qquad a^* = \dfrac{1}{1+\lambda}$

c) $U(x) = x^2$

① $\begin{cases} \max_{\{a\}} & E(U(x)) \\ & U(x) = x^2 \end{cases}$ $E(U(x)) = \frac{1}{2}(2-(1+\lambda)a)^2 + \frac{1}{2}(2a-(1+\lambda)a)^2$

CPO :

② $\frac{dE(U(x))}{da} = 0$

$0 = -(2-(1+\lambda)a)(1+\lambda) + (2a-(1+\lambda)a)(2-(1+\lambda))$

$-(1+\lambda)(2-(1+\lambda)a) + (2a-(1+\lambda)a)(1-\lambda) = 0$

$-(1+\lambda)(2-a-\lambda a) + (2a-a-\lambda a)(1-\lambda) = 0$

$-(1+\lambda)(2-a-\lambda a) + a(1-\lambda)^2 = 0$

$a(1-\lambda)(1-\lambda) - (1+\lambda)(2-a-\lambda a) = 0$

$a(1-\lambda) < 2 - a - \lambda a$

• $\frac{dE(U)}{da} < 0$ $a^* = 0$

d) $U(x) = -e^{-x}$

$E(U) = -\frac{1}{2}e^{-(2-(1+\lambda)a)}$ $\frac{dE(U)}{da} = 0$

$-\frac{1}{2}(1+\lambda)e^{-(2a-(1+\lambda)a)} = 0$

$(1+\lambda)\exp(-2+(1+\lambda)a)$ $-(2)-(1+\lambda)\exp(-2a+(1+\lambda)a) = 0$

$\Leftrightarrow (1+\lambda)\exp(-2+(1+\lambda)a) = (1-\lambda)\exp(-2a+(1+\lambda)a)$

$\Leftrightarrow \frac{1-\lambda}{1+\lambda} = \frac{\exp(-2+(1+\lambda)a)}{\exp(-2a+(1+\lambda)a)}$

$\Leftrightarrow \frac{1-\lambda}{1+\lambda} = \exp(-2+(1+\lambda)a-(-2a+(1+\lambda)a)) = \exp(-2+2a) = \exp(-2(1-a))$

$\Leftrightarrow -\frac{1}{2}\ln\left(\frac{1-\lambda}{1+\lambda}\right) = 1-a \Leftrightarrow a^* = 1+\frac{1}{2}\ln\left(\frac{1-\lambda}{1+\lambda}\right)$

$= 1-\frac{1}{2}\ln\left(\frac{1+\lambda}{1-\lambda}\right)$

EXERCICE 3 :

a) Esperance d'utilité $E(U) = \frac{1}{2}\ln(2+w-(1+\lambda)a) + \frac{1}{2}\ln(2a+w-(1+\lambda)a)$

b) $a^* = ?$ $\begin{cases} \max E(U) \\ \text{sc } U(x) = \ln(x) \end{cases}$ CPO : $\frac{dE(U)}{da} = 0$ $\ln(U)' = \frac{U'}{U}$

$\frac{1}{2}\cdot\frac{-(1+\lambda)}{2+w-(1+\lambda)a} + \frac{1}{2}\cdot\frac{2-(1+\lambda)}{2a+w-(1+\lambda)a} = 0 \Leftrightarrow \frac{2-1-\lambda}{2a+w-(1+\lambda)a} = \frac{1+\lambda}{2+w-(1+\lambda)a}$

$\Leftrightarrow \frac{1-\lambda}{2a+w-(1+\lambda)a} = \frac{1+\lambda}{2+w-(1+\lambda)a} \Leftrightarrow 2(1-\lambda)+w(1-\lambda)-(1+\lambda)(1-\lambda)a = 2a(1+\lambda)+(1+\lambda)w-(1+\lambda)^2$

$\Leftrightarrow -2a(1+\lambda)+(1+\lambda)^2a-(1+\lambda)(1-\lambda)a = (1+\lambda)w-2(1-\lambda)-w(1-\lambda)$

$\Leftrightarrow a[(1+\lambda)^2-2(1+\lambda)-(1+\lambda)(1-\lambda)] = w(1+\lambda-1+\lambda)-2(1-\lambda)$

$\Leftrightarrow a[1+2\lambda+\lambda^2-2-2\lambda-(1-\lambda^2)] = 2\lambda w-2-2\lambda$

$\Leftrightarrow a[-2+2\lambda^2] = 2(\lambda w-1)-\lambda)$

$\Leftrightarrow 2a[\lambda^2-1] = 2(\lambda w-1-\lambda)$ $a = \frac{\lambda w-1-\lambda}{\lambda^2-1}$

$a^* = \frac{1+\lambda-\lambda w}{1-\lambda^2} = \frac{1}{1-\lambda^2} + \frac{\lambda}{1-\lambda^2} + \frac{\lambda w}{1-\lambda^2}$

c) $\frac{da^*}{dw} = \ominus \frac{\overbrace{\lambda}^{>0}}{\underbrace{1-\lambda^2}_{>0}} \Big\} < 0 \quad \forall \lambda \in [0;1[$

EXERCICE 4

alea moral = conseq d'une asymetrie d'info

TD 7

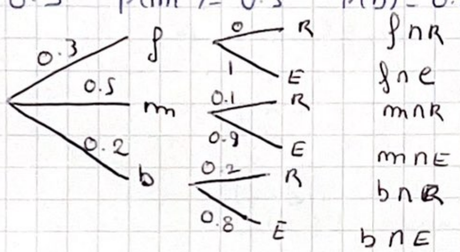
EXERCICE 1 : on cherche $P(f|R)$, $P(m|R)$, $P(b|R)$

$P(R|f) = 0.8$ $\quad P(R|m) = 0.9$ $\quad P(R|b) = 1$

FORMULE DE BAYES : $P(f|R) = \frac{P(R|f)\,P(f)}{P(R|f)P(f) + P(R|m)P(m) + P(R|b)P(b)}$

$= 0.2696$

$P(f) = 0.3$ $\quad P(m) = 0.5$ $\quad P(b) = 0.2$



$P(f|R) = \frac{P(f \cap R)}{P(f\cap R) + P(m \cap R) + P(b \cap R)}$

EXERCICE 2 :

a) aucune informat° $\quad$ R : reforme $\quad \bar{R}$: pas de reforme

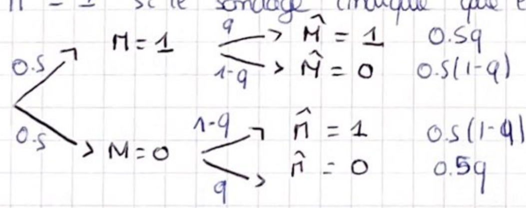
R = 1 si reelection et 0 sinon

$E(R|R) = 0.5 \times 1 + 0.5 \times 0 = 0.5$ $\qquad E(R|\bar{R}) = 0.6 \times 1 + 0.4 \times 0 = 0.6$

b) sondage : $\frac{1}{2} < q < 1$

$M = 1$ si la majorité en faveur (0 sinon)

$\hat{M} = 1$ si le sondage indique que les electeurs sont en faveur (0 sinon)



$P(M=1|\hat{M}=0) = \frac{0.5(1-q)}{0.5(1-q)+0.5q} = 1-q$

↳ $\frac{P(M=1 \cap \hat{M}=0)}{P(\hat{M}=0)}$

$P(M=0|\hat{M}=0) = \frac{0.5q}{0.5(1-q)+0.5q} = q$

Si le sondage revient positif :

$P(M=1|\hat{M}=1) = \frac{0.5q}{0.5q+0.5(1-q)} = q$

$P(M=0|\hat{M}=1) = \frac{0.5(1-q)}{0.5q+0.5(1-q)} = 1-q$